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WAITING LINE AND QUEUEING MODELS

Final Report

Sheldon M. Ross

October 1980



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## ABSTRACT

A large number of waiting line and scheduling models have been considered under the support of the ONR contract N00014-77-C-0299.

# WAITING LINE AND QUEUEING MODELS

Period: November 1, 1976 - April 30, 1980

Our main goal on this project has been to develop theory and approximation for analyzing stochastic waiting line models.

One area of interest was in analyzing such models when the arrival process was nonstationary. In [1] Ross considered a class of single server infinite capacity queueing models in which the arrival process is a nonstationary process with an intensity function $\Lambda(t)$ , $t \geq 0$ , which is itself a random process. He supposed that the average value of the intensity function exists and is equal to some constant, call it $\lambda$ , with probability 1. He then conjectured that the "closer $\{\Lambda(t), t \geq 0\}$ is to the stationary Poisson process with rate $\lambda$" then the smaller is the average customer delay. This conjecture was then verified for the special case where $\{\Lambda(t), t \geq 0\}$ is a 2-state continuous time Markov chain that alternates between values $\lambda_1$ , and $\lambda_2$ , spending a mean time $\alpha_i/c$ in state $\lambda_i$ , $i = 1,2$ . The verification was accomplished by studying how the average delay behaved as a function of $c$ (which regulates how fast the arrival rate is switching between its 2 values). In particular he showed that the average customer delay is a monotone decreasing function of $c$ having as its limit, as $c \to \infty$ , the average delay of the stationary Poisson model having arrival rate $\lambda = \dfrac{\lambda_1\alpha_1 + \lambda_2\alpha_2}{\alpha_1 + \alpha_2}$ .

In a following paper Fond and Ross [2] considered a single server exponential queueing loss system in which the arrival and service rates alternate between the pairs $(\lambda_1,\mu_1)$ and $(\lambda_2,\mu_2)$ , spending an exponential amount of time with rate $c\alpha_i$ in $(\lambda_i,\mu_i)$ , $i = 1,2$ . They showed that if all arrivals finding the server busy are lost then the percentage of arrivals lost is a decreasing function of $c$ . This result is in line with the general conjecture made in the previous paper [1].

In the area of multi-server models, Nozaki and Ross in [3] extended their results on $M/G/k$ queueing models (see [4]) to the situation where the system has a finite capacity $N$. That is they consider a model in which customers, arriving in accordance with a Poisson process having rate $\lambda$, enter the system if there are less than $N$ others present when they arrive, and are then serviced by one of $k$ servers each of whom has service distribution $G$. By means of an approximation assumption, first introduced in their earlier paper [4], they derived an approximation for the average time spent waiting in queue by an entering customer.

Another area considered as that of the optimal scheduling of servers. In [5] Ross (along with his colleagues Derman and Lieberman) considered an $N$ server queuing system in which service times of server $i$ are exponentially distributed random variables with rate $\lambda_i$. Customers are assumed to arrive in accordance with some arbitrary arrival process. If a customer arrives when all servers are busy, then he is lost to the system; otherwise he is assigned to one of the free servers according to some policy. Once a customer is assigned to a server he remains in that status until service is completed. They showed that the policy that always assigns an arrival to that free server whose service rate is largest (smallest) stochastically minimizes (maximizes) the number in the system.

In another scheduling model [6], Pinedo and Ross considered a model in which $n$ jobs have to be performed sequentially in time - the $i^{th}$ job requiring a random time $X_i$ for its execution. In addition, they supposed there are external shocks which occur according to a nonhomogeneous Poisson process. If a shock occurs during the performance of a job then work on that job ends and work on the next one commences. A job is said to be successfully performed if no shocks occur during its execution time. They were interested in determining the job schedule that maximizes the expected number of successful job performances. As a means to determining this, they first considered and solved the related problem of stochastically maximizing the length of time until all jobs are finished (either successfully or by shocks). This related problem is of independent interest for, by interpreting the $n$ jobs as being $n$ spares in a stockpile, it becomes one of stochastically maximizing the life of a stockpile of spares which are subject to shocks which kill any spare in use when the shock occurs.

The above is, of course, only a partial list of the various researches performed under the contract. Further impressions can be obtained by the list of technical reports.

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# PERSONNEL SUPPORTED

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# DEGREES AWARDED

November 1, 1976 - April 30, 1980

Eric Capaldi, M.S.
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Shun-Chen Niu, Ph.D.
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Shirley Nozaki, Ph.D.
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Zvi Schechner, Ph.D.
June 1980